# RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

**B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2018**

**FIRST YEAR [BATCH 2018-21]**

Date : 24/12/2018

**MATHEMATICS (General)**

Time : 11 am – 2 pm

**Paper : I**

Full Marks : 75

## (Use a separate Answer book for each group)

## Group – A

Answer **any five** questions of the following : [5 × 5]

1. a) Find the cube roots of (–1).
   b) If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then prove that $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$. (2+3)

2. Show that $\tan\left(i\log\dfrac{a-ib}{a+ib}\right) = \dfrac{2ab}{a^2-b^2}$.

3. Without expanding in any way, find the simplest value of the following determinant:
$$\begin{vmatrix} 0 & a & b \\ -a & 0 & -c \\ -b & c & 0 \end{vmatrix}.$$

4. Check whether the following system is consistent or not. Solve the system, if consistent.
$$x + y + z = 9$$
$$3x - 2y + 4z = 3$$

5. Find the equation whose roots are the roots of the equation $x^4 - 3x^3 + 2x^2 + 7x - 5 = 0$, each diminished by 2.

6. Solve $x^3 - 15x - 126 = 0$ using Cardan's method.
7. Solve $2x - y = 3,\ 3y - 2z = 5,\ 2z - x = -4$ by Cramer's Rule.

8. Find the values of $\lambda$, for which the equation $x^4 + 4x^3 - 2x^2 - 12x + \lambda = 0$ has four real and unequal roots.

## Group – B

Answer **any five** questions of the follwing : [5 × 5]

9. a) Find a map $f:\mathbb{N}\to\mathbb{N}$ which is one to one but not onto.
   b) Find a map $f:\mathbb{N}\to\mathbb{N}$ which is onto but not one to one. (2+3)

10. a) Prove that $\mathrm{M}=\left\{\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} : a,b,c \in \mathbb{R}\right\}$ forms a subring of the ring of $2\times2$ real matrices under matrix addition and matrix multiplication.

b) Check whether $\mathrm{S}=\left\{\begin{bmatrix} 0 & 0 \\ a & b \end{bmatrix} : a,b \in \mathbb{Z}\right\}$ forms a field under matrix addition and matrix multiplication. (2+3)

11. If $(G,*)$ is a group and $H$ and $K$ are subgroups of $(G,*)$, then show that $H \cup K$ forms a subgroup of $(G,*)$, if either $H \subset K$ or $K \subset H$.

12. Prove that $\mathbb{Q}(\sqrt{2})=\left\{a+b\sqrt{2} : a,b \in \mathbb{Q} \text{ and } (a,b) \neq (0,0)\right\}$ forms a group under multiplication.

13. Show that $W=\{(x,y,z) : x-3y+4z=0\}$ is a subspace of $\mathbb{R}^3$.

14. Find a basis of $\mathbb{R}^3$ containing (1, 1, 2) and (3, 5, 2).

15. Show that the quadratic form $x^2+2y^2+3z^2-2xy+4yz$ is indefinite.

16. Find all the eigen values of the following real matrix:

$$A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Find one eigen vector corresponding to the largest eigen value found above. (2+3)

## Group – C

Answer **any five** questions of the following : [5 × 5]

17. If $u = x\phi\left(y/x\right)+\psi\left(y/x\right)$, then show that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0.$$

$$\left(\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial y\partial x}\right)$$

18. Let $f(x,y)=\begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}, & (x,y)\neq(0,0) \\ 0, & \text{otherwise} \end{cases}$

Show that $f$ is continuous at (0,0).

Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at (0,0).

19. Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=1$ at the point $\left(\frac{1}{4},\frac{1}{4}\right)$.

20. a) Verify Schwarz theorem for $f(x,y)=x^2\tan xy$ at the origin.

b) Is $f(x,y)=\tan^{-1}\frac{y}{x}$ a homogeneous function? If yes, does it satisfy Euler's theorem? Justify. (2+3)

21. Show that the pedal equation of the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ is $r^2+3p^2=a^2$.

22. Show that $\sqrt{7}$ is an irrational number.

23. a) A function $f:\mathbb{R}\to\mathbb{R}$ is defined by

$$f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$$

Show that f is continuous everywhere on $\mathbb{R}$.

b) Let $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=x^3+2x+3,\ x\in\mathbb{R}.$$

Show that $f$ has an inverse (function) $g:\mathbb{R}\to\mathbb{R}$. (2+3)

24. If $x + y = 1$, prove that the $n^{\text{th}}$ derivative of $x^n y^n$ is

$$n!\left\{y^n-\left({}^nc_1\right)^2 y^{n-1}x+\left({}^nc_2\right)^2 y^{n-2}x^2-\left({}^nc_3\right)^2 y^{n-3}x^3+\cdots+(-1)^n x^n\right\}.$$

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